How do develop a numerical project

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Some basic ingredients for a successful numerical project

In when building up a numerical project there are several elements you should think of

1. How to structure a code in terms of functions
2. How to make a module
3. How to read input data flexibly from the command line
4. How to create graphical/web user interfaces
5. How to write unit tests (test functions or doctests)
6. How to refactor code in terms of classes (instead of functions only)
7. How to conduct and automate large-scale numerical experiments
8. How to write scientific reports in various formats (LaTeX, HTML)

Additional benefits: A structure approach to solving problems

The conventions and techniques outlined here will save you a lot of time when you incrementally extend software over time from simpler to more complicated problems. In particular, you will benefit from many good habits:

1. New code is added in a modular fashion to a library (modules)
2. Programs are run through convenient user interfaces
3. It takes one quick command to let all your code undergo heavy testing

4. Tedious manual work with running programs is automated,

5. Your scientific investigations are reproducible, scientific reports with top quality typesetting are produced both for paper and electronic devices.

Analysis of project, Configuration Interaction theory

```
from numpy import *
from sympy import *
from matplotlib.pyplot import *


g_array = linspace(-1, 1, 1001)
e1_array = []
e2_array = []

for g in g_array:
        H1 = matrix([[2-g , -g/2.,  -g/2., -g/2., -g/2.,     0],
                     [-g/2.,   4-g,  -g/2., -g/2.,    0., -g/2.],
                     [-g/2., -g/2.,    6-g,     0, -g/2., -g/2.],
                             [-g/2., -g/2.,      0,   6-g, -g/2., -g/2.],
                             [-g/2.,     0,  -g/2., -g/2.,   8-g, -g/2.],
                             [0    , -g/2.,  -g/2., -g/2., -g/2.,  10-g]])

        H2 = matrix([[2-g , -g/2.,  -g/2., -g/2., -g/2.],
                     [-g/2.,   4-g,  -g/2., -g/2.,    0.],
                     [-g/2., -g/2.,    6-g,     0, -g/2.],
                             [-g/2., -g/2.,      0,   6-g, -g/2.],
                             [-g/2.,     0,  -g/2., -g/2.,   8-g]])



        u1, v1 = linalg.eig(H1)
        u2, v2 = linalg.eig(H2)

        if g == 1./2:
                print argmin(u1)

                for i in range(5):
                        print " %.3f " % v2[i,0],



        e1_array.append(min(u1))
        e2_array.append(min(u2))


plot(g_array, e1_array, linewidth=2.0)
#plot(g_array, e2_array, linewidth=2.0)
plot(g_array, (2-g_array), linewidth=2.0)
grid()
xlabel(r"Strength of interaction, $g$", fontsize=16)
ylabel(r'Ground state energy', fontsize=16)
#axis([-1,1,-0.4,0.05])
legend(['FCI -- Exact', 'Reference energy'])
savefig("proj1_ref2.pdf")
show()
```

Analysis of project, Many-body perturbation theory

```
from sympy import *
from pylab import *

below_fermi = (0,1,2,3)
above_fermi = (4,5,6,7)

states = [(1,1),(1,-1),(2,1),(2,-1),(3,1),(3,-1),(4,1),(4,-1)]
N = 8
g = Symbol('g')



def h0(p,q):
        if p == q:
                p1, s1 = states[p]
                return (p1 - 1)
        else:
                return 0

def f(p,q):
        if p == q:
                return 0

        s = h0(p,q)
        for i in below_fermi:
                s += assym(p,i,q,i)
        return s


def assym(p,q,r,s):
        p1, s1 = states[p]
        p2, s2 = states[q]
        p3, s3 = states[r]
        p4, s4 = states[s]

        if p1 != p2 or p3 != p4:
                return 0
        if s1 == s2 or s3 == s4:
                return 0
        if s1 == s3 and s2 == s4:
                return -g/2.
        if s1 == s4 and s2 == s3:
                return g/2.

def eps(holes, particles):
        E = 0
        for h in holes:
                p, s = states[h]
                E += (p-1)
        for p in particles:
                p, s = states[p]
                E -= (p-1)
        return E


# Diagram 3
# s = 0
```

```
# for a in above_fermi:
#         for b in above_fermi:
#                 for c in above_fermi:
#                         for i in below_fermi:
#                                 for j in below_fermi:
#                                         for k in below_fermi:
#                                                 s += assym(i,j,a,b)*assym(a,c,j,k)*assym(b
# print s


# ga = linspace(-1,1,101)
# corr2 = []
# corr3 = []
# corrx = []


# Diagram 1
s1 = 0
for a in above_fermi:
        for b in above_fermi:
                for i in below_fermi:
                        for j in below_fermi:
                                s1 += 0.25*assym(a,b,i,j)*assym(i,j,a,b)/eps((i,j),(a,b))

# Diagram 4
s4 = 0
for a in above_fermi:
        for b in above_fermi:
                for c in above_fermi:
                        for d in above_fermi:
                                for i in below_fermi:
                                        for j in below_fermi:
                                                s4 += 0.125*assym(i,j,a,b)*assym(a,b,c,d)*as

# Diagram 5
s5 = 0
for a in above_fermi:
        for b in above_fermi:
                for i in below_fermi:
                        for j in below_fermi:
                                for k in below_fermi:
                                        for l in below_fermi:
                                                s5 += 0.125*assym(i,j,a,b)*assym(k,l,i,j)*as

# Diagram 8 (simplified)
s8 = 0
for a in above_fermi:
        for b in above_fermi:
                for i in below_fermi:
                        for j in below_fermi:
                                for k in below_fermi:
                                        s8 -= 0.5*assym(i,j,a,b)*assym(a,b,i,k)*f(k,j)/eps((

# Diagram 9 (simplified)
s9 = 0
for a in above_fermi:
        for b in above_fermi:
                for c in above_fermi:
                        for i in below_fermi:
                                for j in below_fermi:
                                        s9 += 0.5*assym(i,j,a,b)*assym(a,c,i,j)*f(b,c)/eps((
```

```
print s1
print s4
print s5
print s8
print s9

s_5 =  -0.0291521990740741*g**4
s14 =  -0.0308883101851853*g**4
s34 =  0.0163049768518519*g**4
s36 =  -0.0145760995370371*g**4
s38 =  -0.0201099537037037*g**4
s39 =  0.0176938657407407*g**4

ga = linspace(-1,1,10001)
e1 = []
corr2 = []
corr3 = []
corr4 = []
for g_val in ga:
        H1 = matrix([[2-g_val , -g_val/2.,  -g_val/2., -g_val/2., -g_val/2.,     0],
                        [-g_val/2.,   4-g_val,  -g_val/2., -g_val/2.,    0., -g_val/2.],
                        [-g_val/2., -g_val/2.,    6-g_val,     0, -g_val/2., -g_val/2.],
                                [-g_val/2., -g_val/2.,      0,   6-g_val, -g_val/2., -g_val/2.],
                                [-g_val/2.,     0,  -g_val/2., -g_val/2.,   8-g_val, -g_val/2.],
                                [0    , -g_val/2.,  -g_val/2., -g_val/2., -g_val/2.,  10-g_val]])

        u1, v1 = linalg.eig(H1)
        e1.append(min(u1))

        corr2.append((s1).subs(g,g_val))
        corr3.append((s1+s4+s5).subs(g,g_val))
        corr4.append((s1+s4+s5+2*s_5+2*s14+2*s34+2*s36+s38+2*s39).subs(g,g_val))

exact = e1 - (2-ga)

plot(ga, exact, linewidth=2.0)
plot(ga, corr2, linewidth=2.0)
plot(ga, corr3, linewidth=2.0)
plot(ga, corr4, linewidth=2.0)
xlabel(r'Interaction strength, $g$', fontsize=16)
ylabel(r'Correlation energy', fontsize=16)
axis([-1,1,-0.5,0.05])
grid()
legend(["Exact", "2. order MPBT", "3. order MPBT", "4. order MPBT"], 'lower left')
savefig("pert_2.pdf")
show()


error1 = zeros(len(exact))
error2 = zeros(len(exact))
error3 = zeros(len(exact))

for i in range(len(exact)):
        error1[i] = abs(float(exact[i]-corr2[i]))
        error2[i] = abs(float(exact[i]-corr3[i]))
        error3[i] = abs(float(exact[i]-corr4[i]))

error1 = array(error1)
error2 = array(error2)
```

```
error3 = array(error3)
print type(error1)

plot(ga, log10(error1))
plot(ga, log10(error2))
plot(ga, log10(error3))
xlabel(r"Strength of interaction, $g$", fontsize=16)
ylabel(r"Error, $\log_{\rm 10}({\rm abs}({\rm error}))$", fontsize=16)
legend(["2. order MPBT", "3. order MPBT", "4. order MPBT"], 'lower left')
grid()
savefig("logerror.pdf")
show()
```